MATH4120: Mathematical Modelling and Programming

Exercise sheet 3: Solving ordinary differential equations

Chapter 4: separable equations, integrating factors, Bernoulli and homogeneous substitutions, second-order linear ODEs

SUBMISSION DEADLINE
Friday 6 November 2026, 16:00

Submit a single scanned PDF via Moodle, please show all of your working.

Question 1 [9 marks]

For each of the following, first classify the equation (separable, linear, Bernoulli or homogeneous), then solve it by the appropriate method.

(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = x\,e^{-y}$, with $y(0) = 0$.

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2}{x}\,y = 5x^2$, for $x > 0$.

(c) $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x} = x\,y^3$, for $x > 0$.

Question 2 [5 marks]

Solve the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2 + 3y^2}{2xy}, \qquad y(1) = 1,$$

for $x > 0$. Give your answer as a relation between x and y, and state the explicit solution for y.

Question 3 [6 marks]

Consider the second-order equation

$$y'' - 4y' + 3y = 6e^{3x}.$$

(a) Find the complementary function.

(b) Explain why the trial particular integral $\phi = Ae^{3x}$ *cannot* work, and state the correct trial form.

(c) Find the particular integral and hence the general solution.

(d) Find the solution satisfying $y(0) = 0$ and $y'(0) = 1$.