MATH4120: Mathematical Modelling and Programming

Exercise sheet 2: Scaling and dimensional analysis

Chapters 2–3: nondimensionalisation, solution collapse, the Buckingham Π-theorem and the dimension matrix

SUBMISSION DEADLINE
Friday 30 October 2026, 16:00

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Question 1 [5 marks]

A drug is infused into a patient at a constant rate D ($\mathrm{mg\,hr^{-1}}$) and eliminated at a rate proportional to its concentration, so that the concentration $C(t)$ ($\mathrm{mg\,L^{-1}}$) satisfies

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{D}{V} - kC, \qquad C(0) = 0,$$

where V is the volume of distribution (L) and k the elimination rate ($\mathrm{hr^{-1}}$).

(a) Identify a natural concentration scale and a natural time scale for this problem.

(b) Using these scales, show that the model reduces to a parameter-free initial value problem, and solve it.

(c) Explain what *solution collapse* means here: what happens if two patients have different D, V and k?

Question 2 [5 marks]

The rate at which an enzyme consumes its substrate is often modelled by Michaelis–Menten kinetics,

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{V_{\max} S}{K_m + S}, \qquad S(0) = S_0,$$

where $S(t)$ is the substrate concentration, $V_{\max}$ is the maximum reaction rate and K_m is a reference concentration.

(a) Determine the dimensions of $V_{\max}$ and K_m.

(b) Nondimensionalise using $s = S/K_m$ and a suitable dimensionless time τ, stating your choice of time scale.

(c) Show that exactly one dimensionless parameter remains, and interpret it.

Question 3 [6 marks]

The fundamental frequency f of a vibrating string is thought to depend on its length L, the tension T in the string, and its mass per unit length ρ_ℓ.

(a) Write down the dimensions of f, L, T and ρ_ℓ.

(b) How many dimensionless groups does the Π-theorem predict?

(c) Construct the dimension matrix, solve for the exponents, and hence show that $f = \dfrac{C}{L}\sqrt{\dfrac{T}{\rho_\ell}}$ for some dimensionless constant C.

(d) A guitarist wants to double the frequency of a string without changing its length or material. By what factor must the tension change?

Question 4 [4 marks]

An engineer tests a 1:20 scale model of a car in a wind tunnel. The tunnel uses air at the same temperature and pressure as the road, so ρ and μ are unchanged.

(a) State the condition for dynamic similarity between the model and the full-size car, and explain why it guarantees the same drag coefficient.

(b) Find the wind speed required in the tunnel to reproduce the flow around a full-size car travelling at $30\ \mathrm{m\,s^{-1}}$.

(c) Comment on whether this is practical. The speed of sound in air is about $343\ \mathrm{m\,s^{-1}}$.